Salisbury University Department of Physics

DERIVATION OF EQUATION FOR MOMENT OF INERTIA

SU PHYSICS

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Conservation of Energy – Experimental Apparatus w/ Core

Physical Concept: Conservation of Energy: Energy cannot be created or destroyed; it may be transformed from one form into another, but the total amount of energy never changes.

Energy is one of the most important concepts in physics. Yet it is not an easy concept to describe simply. There are many different kinds of energy any of which can be described. In any process we find that the total energy of the system never changes. It is conserved. However, energy can change from one form to another.

Gravitational Potential Energy: An object of mass may store energy by virtue of its position relative to the gravitational tug of the Earth. Such stored energy is called potential energy because the energy is describing the potential of the object to do work due to the force of gravity. Gravitational Potential Energy can be described by the following equation:

$$\text{P.E.} = mgh \qquad (1)$$

where "m" is the mass of the object, "g" the acceleration of gravity (of Earth), and "h" is the height of the mass.

If I raise a bowling ball above the ground it has the potential to do work. If I drop it, I could drive a nail. The object has the potential to do work due to gravity. There is no absolute scale for potential energy. We always measure the height with respect to some reference point. For instance, if I define the gravitational potential energy to be 0.0 meters on the alley surface, then I raise the bowling ball above that level - it has a potential energy of "*mgh*" relative to the alley surface. Something that is higher has a higher potential energy than something that is lower.

Kinetic Energy of Motion: An object of mass has energy related to the motion of the object. We call this energy kinetic energy. An object in motion has kinetic energy, whether that motion is translational (straight line motion), rotational (object is spinning about an axis), or a combination of the two motions. Kinetic energy is an expression of the fact that a moving object can do work on anything it hits; it quantifies the amount of work the object could do as a result of its motion.

Translational Kinetic Energy can be described by the following equation:

$$\text{K.E.} = \frac{1}{2}mv^2 \qquad (2)$$

the translational kinetic energy of an object is equal to half the object's mass, "m," multiplied by the object's velocity squared, "v^2." Many bowlers are intuitively aware that a bowling ball traveling at 20 miles per hour has four times as much straight line kinetic energy as a ball traveling 10 miles per hour. This means that the faster ball traveling twice as fast has the ability to transfer four times more energy to the pins.

Rotational Kinetic Energy can be described by the following equation:

$$K.E._{rot} = \frac{1}{2} I \omega^2 \tag{3}$$

the rotational kinetic energy of an object is equal to half the object's moment of inertia, "I," multiplied by the object's angular velocity squared, "ω^2." The moment of inertia, "I," is a quantity that describes the rotational inertia of a body which depends on the distribution of mass about the axis of rotation --- the greater the distance between the bulk of an object's mass and its axis of rotation the greater the rotational inertia of that object. Notice that kinetic energy of rotation depends not only on angular velocity but also on the distribution of the mass in the rotating object (i.e. the core shape and mass distribution of a bowling ball will affect the rotational energy of a bowling ball). The rotational inertia (& energy distribution once in motion) of a bowling ball can be varied with different design configurations.

Experimental Apparatus Description and Mathematical Derivation:

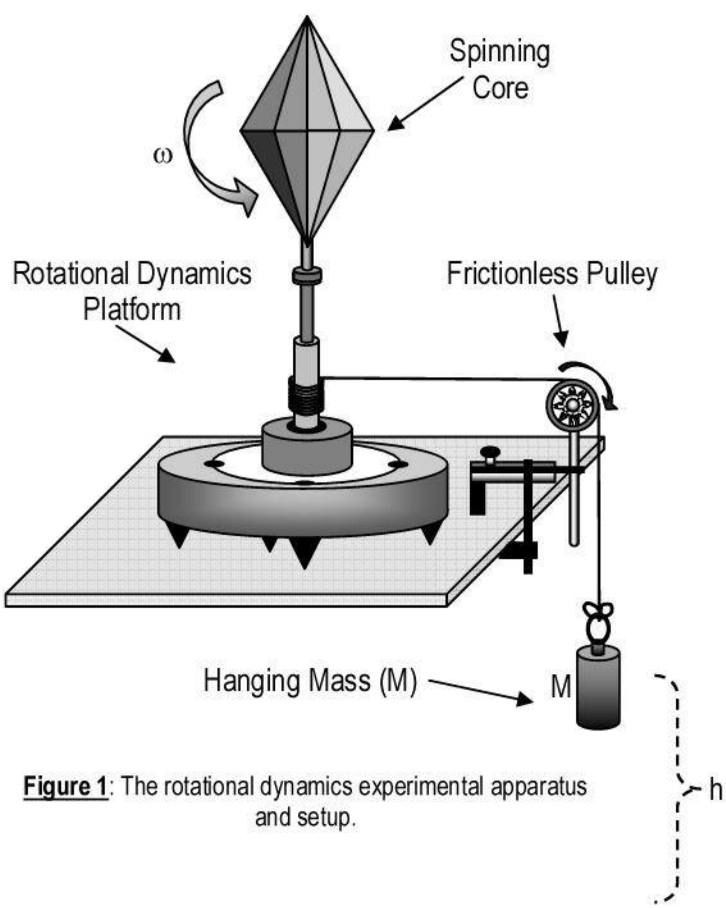

Figure 1: The rotational dynamics experimental apparatus and setup.

The experimental apparatus (Figure 1) consists of a rotational platform upon which different massive objects (ball cores) can be attached and rotated about any fixed axis. A known hanging mass can apply a constant gravitational force to the dynamics platform and cause the object and platform to experience a known torque thus creating measurable rotation from which the principle of energy conservation can be used to determine the object's rotational inertia (also called moment of inertia).

Images of the actual setup as it appeared in the laboratory are shown in figure 2. The core can be placed to rotate about any axis while measurements of the hanging mass, rotational axis, core mass, core shape & size, time duration of the motion of the core and hanging mass, and distance the hanging mass moved were all recorded.

Conservation of energy will yield the following conceptual relationship between energy types;

“the total amount of gravitational potential energy before any motion begins will be transformed into the rotational kinetic energy of the rotational dynamics platform and core plus the translational kinetic energy of the falling hanging mass as the motion takes place.”

The mathematical equation describing this physical concept is as follows;

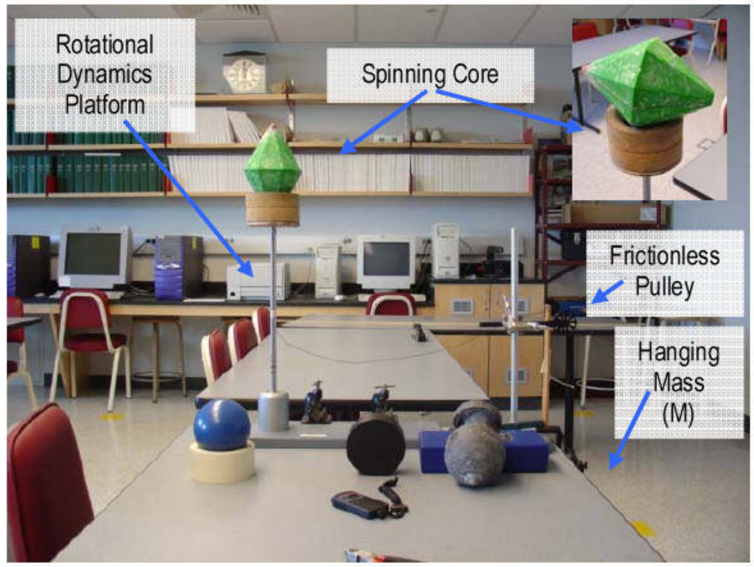

Figure 2: The rotational experimental apparatus.

Gravitational P. E. of Hanging Mass = Translational K.E. of Hanging Mass + Rotational K.E.

$$Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}I_o\omega_f^2 \qquad (4)$$

Now, since the acceleration due to gravity ($g = 9.8\,{}^{m}/_{s^2}$) is a constant we can say the following in terms of the hanging mass final velocity;

$$\text{Avg. } \overline{\text{V}} = \frac{\text{Initial V}_\text{i} + \text{Final V}_\text{f}}{2} = \frac{0\,{}^{m}/_{s} + v_f}{2} = \frac{v_f}{2}$$

also

$$\text{Avg. } \overline{\text{V}} = \frac{\text{height of fall}}{\text{time}} = \frac{h}{t}$$

then

$$v_f = \frac{2h}{t} \qquad (5)$$

In addition, angular velocity of the rotational object is related to the straight line translational velocity via the expression;

$$v = \omega r_p$$

which yields

$$\omega_f = \frac{v_f}{r_p} \qquad (6)$$

Placing equations (5) and (6) into the conservation equation (4) gives a form of the equation that has easily measurable quantities to perform the experiment.

$$Mgh = \frac{1}{2}M\left(\frac{2h}{t}\right)^2 + \frac{1}{2}I_o\left(\frac{v_f}{r_p}\right)^2 \qquad (7)$$

M = hanging mass
h = distance M falls
t = time M falls
I_o = Moment of Inertia of rotating body
v_f = final velocity of M
r_p = radius of dynamics platform pole

Equation (7) can be rearranged algebraically to solve for the rotational inertia term "I_o."

$$I_o = \frac{M r_p^2\left(gt^2 - 2h\right)}{2h} \qquad (8)$$

Equation (8) is the resulting final conservation equation that is used to find the moment of inertial for the ball cores on the experimental apparatus. Careful consideration must be used, however. The equation takes into account any rotational object on the platform as well as the platform itself. So, the experiment is run first with no object rotating, only the platform in rotational motion – this will determine the moment of inertia of the platform alone. Once the ball core is attached to the platform and the experiment is run again, equation (8) will determine the moment of inertia of the platform AND the core. To determine the rotational moment of inertia of just the bowling ball core we will finally use equation (9).

$$I_{core} = I_{core+platform} - I_{platform} \qquad (9)$$